

## COMPARATIVE ANALYSIS OF THE LUNAR POLES USING FRACTAL DIMENSION

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**Abstract:** Surface roughness is one of the fundamental quantitative parameters of digital terrain analysis (DTA). The development of computer systems over recent decades has led to the development and successful implementation of various digital analysis methods. An important place among them is undoubtedly the digital fractal analysis. Along with this, the ever-improving digital elevation models (DEMs) of the Moon's topography provide new opportunities in this direction. Present study focuses on differences in the topography of lunar poles based on the digital fractal approach. For this purpose, the real physical surface of the Moon's poles is represented as "fractal surface", and the differences in hypsometry and surface roughness are described by fractal dimension. The results obtained on this basis showed a different geological history of the lunar poles. This requires a thorough further interpretation.

## СРАВНИТЕЛЕН АНАЛИЗ НА ЛУННИТЕ ПОЛЮСИ ИЗПОЛЗВАЙКИ ФРАКТАЛНА ДИМЕНСИЯ

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**Ключови думи:** Луна, фрактална повърхнина, ЦМР, лунни полюси, ДАР, грапавост

**Резюме:** Грапавостта на релефа е един от фундаменталните качествени параметри на дигиталния анализ на релефа (ДАР). Развитието на компютърните системи през последните десетилетия доведе до развиването и успешното внедряване на различни дигитални методи за анализ. Важно място сред тях без съмнение има дигиталния фрактален анализ. Наред с това, непрекъснато подобряващите се цифрови модели на релефа (ЦМР) за топографията на Луната предоставят нови възможности в тази посока. Настоящото изследване се фокусира върху различията в топографията на лунните полюси базирани на дигиталния фрактален подход. За тази цел, физическата повърхност на лунните полюси е представена като „фрактална повърхнина”, а различията в хипсометрията и грапавостта на релефа са описани чрез фрактална дименсия. Получените резултати на тази основа показват различна геоложка история на лунните полюси. Това налага допълнителна интерпретация.

### Introduction

Over the last decades, the increasingly use of fractal analysis on the one hand and the continuous improvement of digital elevation models (DEMs) have led to the introduction of an innovative methodological approach. The cohesive, but unambiguous, use of fractals and DEMs gave to the scientific community new possibilities for analysis and interpretation of the terrain. The fractal approach is based on the observation that the morphology of surfaces is statistically self-affine, which implies that when repeatedly magnified, increasing details of roughness emerge and appear similar to

the original profile (Taud and Parrot, 2009). With the fractal approach, it is possible to calculate the scale-independent parameters which describe the surface. The fractal measure parameter, i.e. fractal dimension (D), is a well-known measure unit of surface roughness (Mandelbrot, 1982; Pentland, 1984; Franceschetti et al., 2000; Pant et al., 2010; Sun et al., 2006) and represents the capacity of the surface to fill in the adjacent volume (Zahouani et al., 1998). Fractal dimension (FD) has many applications in remote sensing research including image processing, image analysis, texture segmentation, shape classification and identifying the image features such as roughness and smoothness (Nayak and Mishra, 2016).

In recent years, the "fractal approach" has been increasingly applied to the surface of Earth's natural satellite - the Moon. Using fractal analysis, spatial variations and peculiarities of the Moon's topography (Turcotte, 1987; Nefedjev, 2003; Baldassarri et al., 2008; Huang et al., 2009; Rosenberg et al., 2011; Cao et al., 2015; Bray et al., 2017) have been successfully analyzed. Also from the position of the fractals was analyzed and interpreted the Moon's gravity field and its relation to the terrain (Kumar et al., 2016; Ranguelov et al., 2019).

The present study focuses on spatial differences in elevation and surface roughness of the lunar poles. Differences in topography of the poles are analyzed and interpreted by constructing of 2D fractal models (surfaces) based on high resolution DEM data (30x30 meters) from the Lunar Orbiter Laser Altimeter (LOLA) (Smith et al., 2011). The results obtained in the course of the study showed significant differences between lunar poles regarding of topography and its roughness. Unlike other large planetary bodies in the inner solar system, the lunar South Pole's topography is largely redesigned from free space objects. This requires the need for further interpretation.

## Methods and Data

### *Variogram method for fractal dimension estimation*

There are many techniques to estimate the fractal dimension. In the present study the fractal dimension is calculated using Focal Fractal Dimension Calculator (FocalD) based on the „Variogram method” (Mark and Aronson, 1984). The software calculates a surface of fractal dimension values in a window around each raster cell. The pixel signal value in each fractal image reflects the complexity of the variation in the topography. The result is an entire raster map (2D) of fractal dimension values indicating how data changes over space. The fractal estimator (Jaggi et al., 1993) measures fractal dimension (D) based on the variogram computed for the study area, and

$$(1) \quad \gamma(h) = \text{Var}(Z_i - Z_j)$$

where  $i, j$  are spaced by the distance vector  $h$ .

The fractal dimension (D) can be derived by regressing the logarithm of the distance vector with the logarithm of the variance (Zhou and Lam, 2005), and

$$(2) \quad D = 3 - (B/2)$$

where D is fractal dimension and B is the slope of the regression.

The fractal signal value is much higher, when the DEM values have a more complex variation. For example, fractal dimension of 2,0 is an indicator for smooth, scale invariant surface, while fractal dimension of 3,0 is an indicator for a space-filling extremely rough surface (Table 1).

Table 1. Surface roughness classification based on fractal dimension (based on Mark and Aronson, 1984; Pentland, 1984; with modifications)

Class	Surface type	Fractal Dimension (FD)
1	Flat	2,0-2,1
2	Nearly flat	2,1-2,2
3	Slightly rough	2,2-2,3
4	Moderately rough	2,3-2,5
5	Highly rough	2,5-2,8
6	Extremely rough	2,8-3,0

### *Data and software*

The digital elevation model (DEM) of the lunar poles using in the present study is based on data from the Lunar Orbiter Laser Altimeter (LOLA) (Smith et al., 2011), an instrument on NASA's Lunar Reconnaissance Orbiter (LRO) spacecraft (Tooley et al., 2010). The DEM is generated in Projected Coordinate System Moon 2000. The data are available in Georeferenced Tagged Image File Format (GeoTIFF) at 30x30 m spatial resolution.

The DEM data have been processed and explored using Geographic Information System (GIS) - SAGA-GIS (Conrad et al., 2015), QGIS (Thiede et al., 2014) and LandSerf (Wood, 2009) free software.

### Results and Discussion

The results of the study are presented visually and textually in Fig. 1 and Table 2. The main conclusions and interpretations are discussed further.

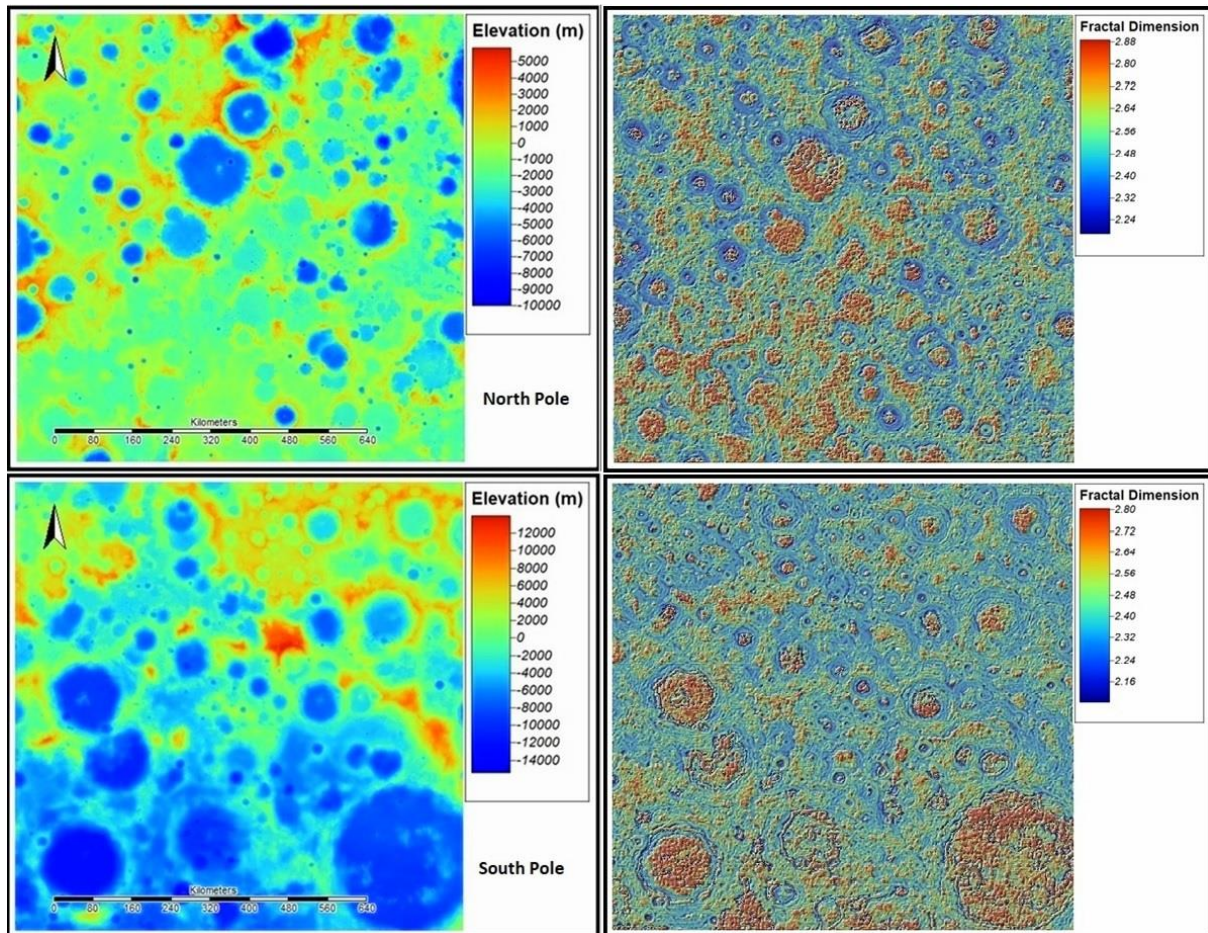


Fig. 1. Lunar poles topography and corresponding 2D fractal surface

Table 2. Topography peculiarities of the lunar poles

<b>North Pole</b>			
DEM min	-10009	FD min	2,08
DEM max	5824	FD max	2,88
SD	4593	SD	0,306
R <sup>2</sup>	0,909	R <sup>2</sup>	0,739
<b>South Pole</b>			
DEM min	-15461	FD min	2,08
DEM max	13968	FD max	2,80
SD	8538	SD	0,209
R <sup>2</sup>	0,944	R <sup>2</sup>	0,733

The presented results show an interesting picture. In general, the terrain of the lunar poles is characterized by a large amplitude regarding to the hypsometry. For the North Pole it is 15 833 m and for the south one 29 429 m. The difference is also significant regarding to the maximum and minimum absolute hypsometry values in favor of the southern lunar pole (Fig. 2). It is remarkable that both Poles have approximately similar values of the FD and R<sup>2</sup>.

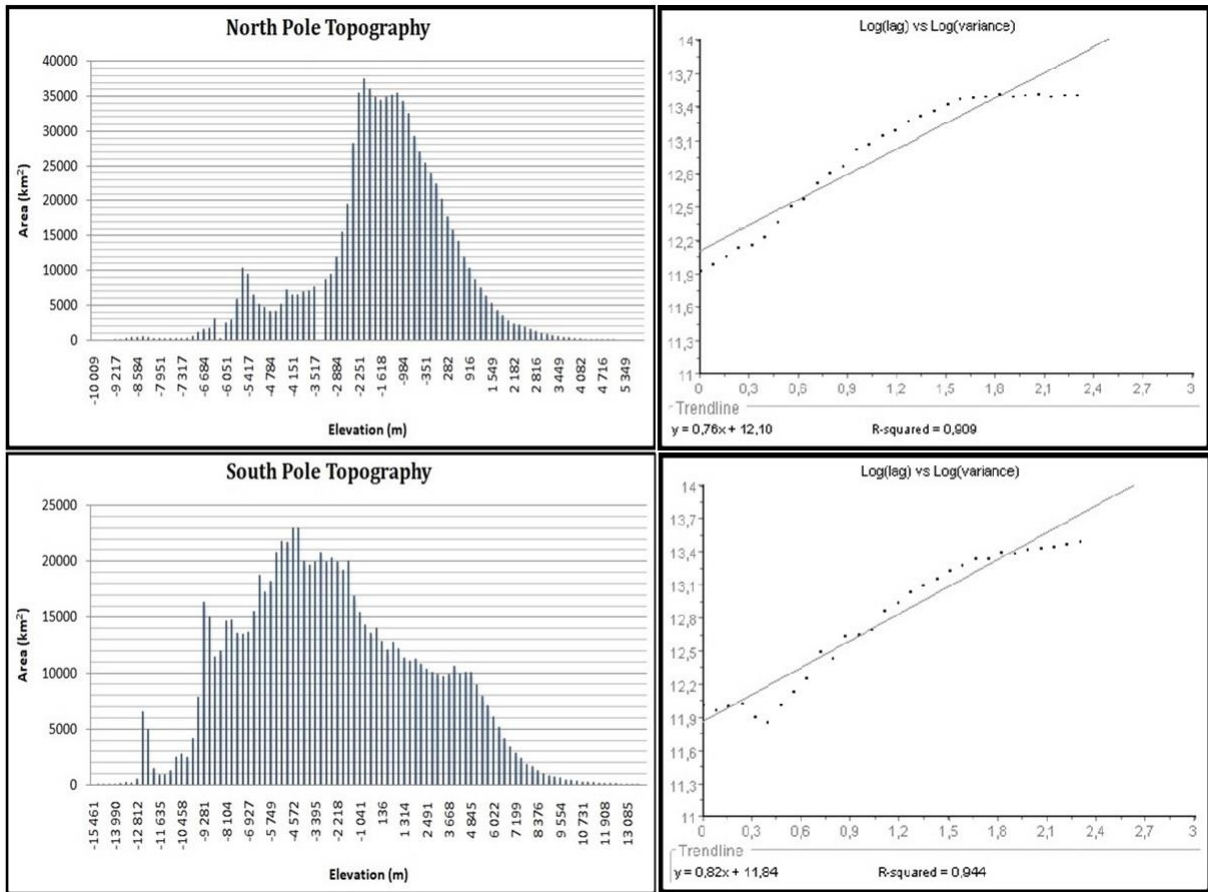


Fig. 2. Lunar poles hypsometry variation

Differences, however, are observed in the spatial distribution of fractal values and related parameters. The South Pole of the Moon ( $R^2 = 0.733$ ) is characterized by a slightly nonlinear distribution of the topography values in comparison of the northern one ( $R^2 = 0.739$ ) (Table 2). In general, based on fractal dimensions (Fig. 3, Table 3), the surface of both poles is characterized by moderately-highly roughness. Within the North Pole, the areas with highly and extremely highly terrain are more widespread (53,4 % > 31 % of total area or 184 124 km<sup>2</sup> difference). The flat areas are rare as a whole, but more widespread within the South Pole (5,5 % > 1,5% of total area or 31 190 km<sup>2</sup> difference). This proves that, compared to the northern pole, the terrain of the southern one is transformed by geological processes with higher power but lower intensity.



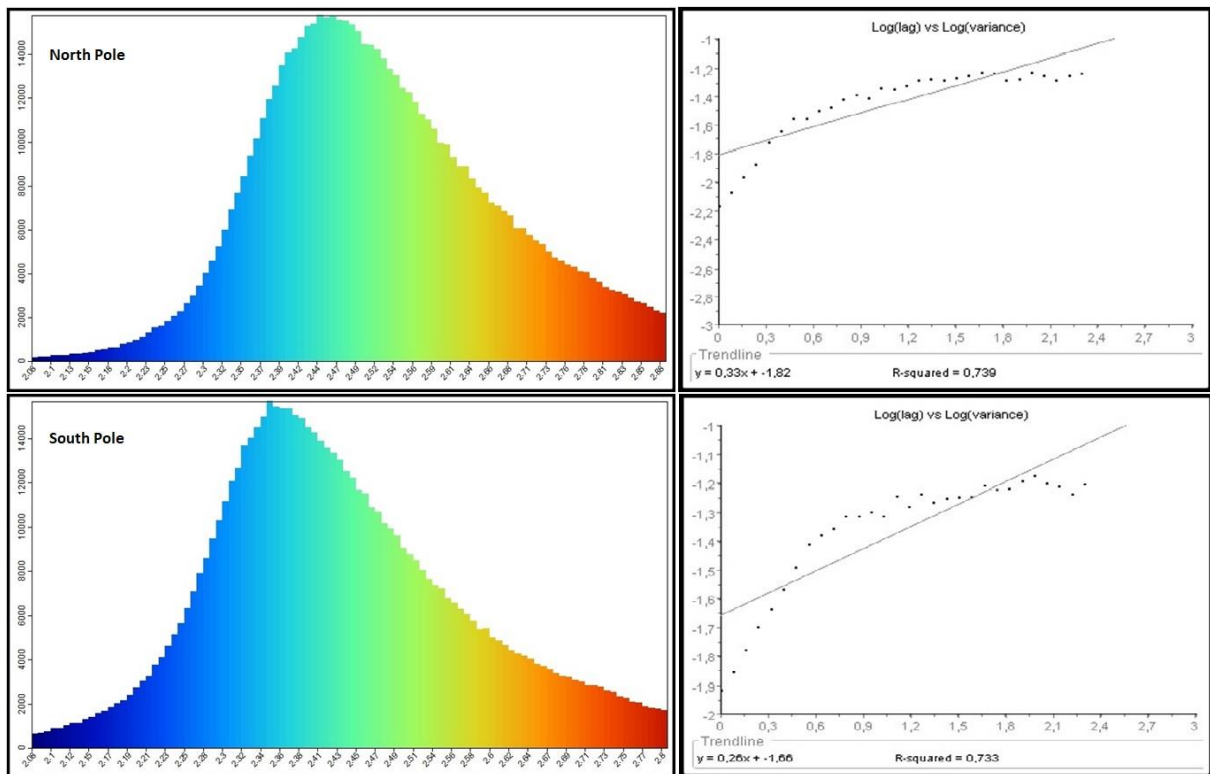


Fig. 3. Frequency of distribution of fractal dimensions within the lunar poles

Table 3. Surface roughness of the lunar poles

Class	Surface type	Fractal Dimension (FD)	North Pole		South Pole	
			Total area (km <sup>2</sup> )	Relative share (%)	Total area (km <sup>2</sup> )	Relative share (%)
1	Flat	2,0-2,1	5380	0,7	20532	2,5
2	Nearly flat	2,1-2,2	6600	0,8	24638	3,0
3	Slightly rough	2,2-2,3	28039	3,4	98554	12,0
5	Moderately rough	2,3-2,5	342544	41,7	422961	51,5
6	Highly rough	2,5-2,8	366249	44,6	216818	26,4
7	Extremely rough	2,8-3,0	72472	8,8	37779	4,6

## Conclusion

The results obtained confirmed the differences in the topography of the lunar poles. From one side the both poles are dominated by the “moderate and highly rough” surfaces. (77.9% for the South Pole and 82.3 % for the North). From the other side - the surface of the southern lunar pole is less expressive but more variable, while within the northern one vice versa- more expressive and less variable. This leads to the conclusion that, compared to the North Pole, the southern one has been subjected to different by intensity and power impact events, which have created and shaped the contemporary pattern of the relief. The fractal analysis clearly confirm that conclusions. This is in contrary to the tendency, the northern hemispheres (and respectively, the poles) of the planetary bodies within the inner solar system to be more vulnerable to collisions with large space objects. This necessitates further in-depth research.

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